

## Truth IV: Kripke

Tarski insisted on a family of languages, each containing a truth predicate for those below it. So if one wants to know whether a given sentence containing a truth predicate is true or false (and in many cases, if one wants to know if it is even grammatical) one will need to know which language it is in.

This is highly unnatural. Further, as Kripke points out, it isn't really workable. The problem comes from the fact that paradox is not generated just by self-reference but also by reference to other sentences. Consider again the two sentences

- (1) Sentence (2) is true
- (2) Sentence (1) is false.

How does Tarski solve this? By insisting that the truth predicates cannot both be in a language higher than the other, so that at least one of the sentences must be paradoxical.

But suppose that we are in an argument, and I shout at you:

Hardly anything that you have said is true

and you shout back

Hardly anything that you have said is true

We will both want our truth predicates to be higher than the others, since whoever gets the higher will render their opponent's claim ungrammatical. So we might each try to make ours higher (by muttering a level under our breath?). But that's ridiculous. And moreover, intuitively it seems that both of our assertions could be true or false; perhaps we had both been lying almost all of the time.

Or consider Kripke's own example.

Dean says: Most of Nixon's Watergate sentences are false

Nixon says: Most of Dean's Watergate sentences are true

Tarski would say that one of these must be ungrammatical, but in most circumstances they would give no risk of paradox; indeed they might easily be true. They would only give rise to paradox in exceptional circumstances: suppose up till now exactly 50% of each of their Watergate sentences were true, and 50% false. Then they would be like (1) and (2).

## KRIPKE'S ACCOUNT

Kripke's main idea is that we can stick with a single truth predicate, if we let each of the utterances find its own level by an inductive procedure. Rather than having a hierarchy of languages, we can have a hierarchy of levels, at each of which we can add instances of the truth predicate. But it is the same truth predicate at each level. Moreover, if we used an axiomatic approach, rather than defining truth explicitly, we do not need a distinction between the object language and the metalanguage: the language can contain its own truth predicate. (The need for an axiomatic approach here comes from the fact, shown by Tarski, that any moderately powerful language that tries to define its own truth predicate will be inconsistent. Kripke himself did give a definition of truth, and so worked with a separate object language and metalanguage, but there are axiomatic developments of his approach that do not; for discussion see Volker Halbach's piece on axiomatic theories of truth in the *Stanford Encyclopedia*). To keep things simple, first consider just the atomic sentences; we'll come to the compound sentences shortly.

Start with the class of all the true atomic declarative sentences that don't contain a truth predicate (or, since that might seem to beg the question, follow Kripke and put it as the class of such sentences that you would be prepared to assert). Then extend that class by adding to it each sentence formed by predicating truth of each of the sentences it already contains; extend it again by predicating truth of those sentences; and so on. Do a parallel thing with the atomic sentences that don't contain a truth predicate that you would be prepared to deny: make a set of them, predicate truth of them and add that to the set, and so on. Then we can make moves between the two classes: predicate falsity of a sentence in one class and then add that to the other (alternatively predicate not-truth: we make no distinction between being false and being not true). Once a sentence has been added to either of the classes in this way, it will of course remain there no matter how many other sentences are added. And provided that an initial class of atomic sentences was consistent (as we might hope is the case with the true ones) it will remain consistent

Crucial to Kripke's account is the idea that there is a *minimal fixed point*: a point at which there are no new sentences to be added to either of the two classes. (Kripke uses the term to refer to the *pair of classes* when they reach this point.) Of course this won't happen at any finite level: for a sentence containing any finite number of truth predicates, you can always add another. But what Kripke proves, using machinery from set theory, that if you go to transfinite levels (representing levels by numbers, these are levels that are represented by numbers larger than any of the natural numbers) then we must arrive at a point at which this the class doesn't grow. At this point the paradoxical sentences like (1) and (2), and also (3):

(3) Sentence (3) is not true

will not be in either of the classes, nor will they occur in either of the classes with truth or falsity predicated of them. The same will hold for 'truth teller' sentences like

(4) Sentence (4) is true

All such sentences will be, in Kripke's term *ungrounded*.

We won't get into trouble if we add (4) to the class of true sentences of our minimal fixed point, and then predicate truth of it: that will give rise to a new fixed point, though the addition may seem arbitrary, since we could equally add it to the class of false sentences. In contrast, if we try to add (3), we won't arrive at another fixed point.

The effect of this will be to effectively define an extension for the truth predicate (all of the sentences that occur in the minimal fixed point with truth predicated of them), and an anti-extension for the truth predicate (all of those sentences that have falsity or not-truth predicated of them). But not every grammatically well-formed sentence occurs in one or other of the classes.

(If we think that there are independent reasons for thinking that some sentences are not truth-apt—as a result of presuppositional failure etc., as discussed last time—we might put them into the third class right from the start; however, Kripke doesn't discuss this.)

The truth predicate thus ends up as a 'gappy' predicate. Following Soames, we might compare it to an artificial predicate like 'magnaped' whose extension and anti-extension are fixed by the following definition (or by ostensively pointing at groups of people who constitute the extension and anti-extension):

Someone who has a shoe size of 11 or greater is a magnaped;  
 Someone who has a shoe size of less than 10 is not a magnaped.

What can we say about someone who has a shoe size of 10.5? Clearly they aren't in either the extension or the anti-extension. But can we say that they are neither a magnaped nor not a magnaped? Apparently not without fear of contradiction. The same applies to truth.

How do we now handle the non-atomic sentences. Once we think of the truth predicate as a partial predicate, we have two possibilities. We might use a three-valued logic, or we might use a supervaluational approach. Kripke focuses on the first, using strong Kleene tables, but briefly discusses the second.

The Strong Kleene three-valued truth-table for disjunction

v	T	–	F
T	T	T	T
–	T	–	–
F	T	–	F

for negation:

$\sim$	
T	F
–	–
F	T

and for conjunction:

$\&$	T	–	F
T	T	–	F
–	–	–	F
F	F	F	F